Scaling angles and distances to maximize efficiency of image registration

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Abstract. Medical image registration is performed by numerical optimization of the transformation parameters including both angular and distance components. These are measured in different units, and as a result a scale factor between them must be used. We propose a scale factor that delivers an improved number of iterations, failure rate and registration error on MRI, CT and scene images.

There are numerous reliable algorithms for rigid registration of 2-D and 3-D images. However, time-critical applications such as interoperative image guidance require the best running time performance possible. Nearly all registration approaches use a numerical optimization framework to find the parameters of a transformation (for reviews, see [1, 2]). The optimization algorithms minimize the metric between a fixed and transformed image by taking a series of steps in the parameter space. Rigid registrations are parameterized with both angular and translational components, which are not directly comparable. For example, a rotation of 1 radian will cause much more change to an image than a shift of one pixel. Therefore most implementations multiply angular parameters by a scale factor, F, between the two to correct for this mismatch.

It is generally agreed that the scale factor is necessary but its effect on performance, and methods for selecting it, have not been extensively investigated. While most optimization approaches will work within a range of scale factors, our preliminary work shows there is a distinct improvement in efficiency gained by setting it correctly.

1 Selecting an optimal scale factor

For this investigation, we used the popular gradient descent optimizers in ITK[1]. These optimizers rely on the gradient being representative of the function shape. For this to be true the second derivative term of a Taylor series approximation cannot be too large. Therefore, F should control the size of the second derivative term. Specifically, F should make the mean squared pixel displacement equal to one, for a scaled angular step of 1. For a rotation about the image center, $\bar{x}_c$, F will be:

$$F = \frac{1}{number\_of\_pixels} \cdot \sum_{\bar{x} \in all\_pixels} (\bar{x} - \bar{x}_c)^2 \tag{1}$$

(For small images, this agrees well with an existing heuristic that F should be 10 times the diagonal length of the image[3]).

2 Experimental Results

We compared this scale factor against factors of different orders of magnitude by rigid registration of images with themselves. Four 2D (MRI, CT, scene images) and three 3D images (MRI, CT) were registered from an average of 10 different starting positions using both mean squared difference and mutual information metrics.

Table 1. Registration Results: Average Of All Cases

(The number of iterations is relative to the iterations,X, when using our factor, F)

Factor:	$F \cdot 10^2$	$F \cdot 10$	F	$F \cdot 10^{-1}$	$F \cdot 10^{-2}$
Iterations Required	3.6X	1.8X	1.0X	3.8X	3.2X
Rotation error:	0.009°	0.002°	0.002°	0.002°	0.012°
Translation error:	0.121 pixels	0.003 pixels	0.002 pixels	0.038 pixels	0.094 pixel
Failure rate:	14.5%	0.8%	0.8%	14.4%	77.1%

The results (summarized in Table 1) show that registrations with F are significantly faster, more reliable, and slightly more accurate. Interestingly, it appears that if in doubt it is better to use a factor which is too large, rather than one which is too small.

3 Conclusion

The angle to distance scale factor, F, significantly influences the performance of image registration. We have derived a method for setting F automatically from image properties that performs well in terms of number of iterations, failure rate and registration error.

References

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